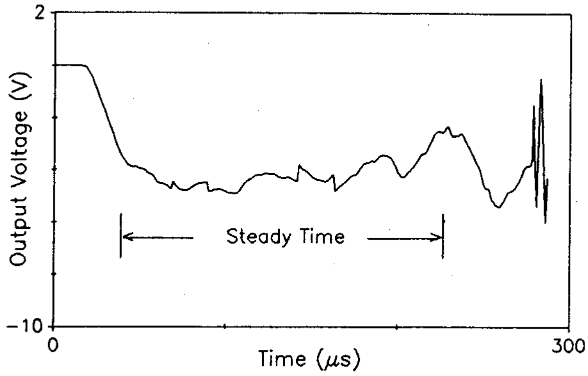

Fig. 2 Uncalibrated time history of filtered gauge output for the following test section conditions: Mach number = 3.1; velocity = 3486 m/s; static pressure = 114 kPa; stagnation enthalpy = 12.1 MJ/kg; temperature = 3383 K.

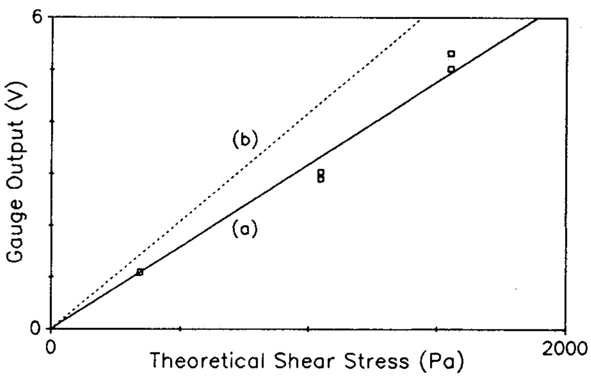

Fig. 3 Measured gauge output in volts (line a) and predicted gauge output (line b) vs theoretical shear stress for test section stagnation pressure from 0.77 to 5.88 MPa; line b is determined from nominal manufacturer's sensitivities for the piezoceramics.

were associated with the flow and not with the gauge, a commercial piezoelectric pressure transducer was mounted in a second flat-plate model to measure static pressure on the surface. The second flat-plate model had substantially different geometry and different flow-induced stress wave reflection times, but the onset of large fluctuations occurred in the unfiltered outputs from both the pressure transducer and one piezoceramic element at the same time, about 250 μs after the start of the flow. We conclude that the large-scale unsteadiness after 250 μs is due either to shock wave reflections resulting from the impulsive flow meeting the back face of the dump tank or to the arrival of the helium driver gas. However, in the available 200 μs of useful test time, the skin-friction gauge indicated steady boundary-layer flow (Fig. 2).

Figure 3 displays the skin-friction gauge averaged output voltage in the 200-μs test time plotted against theoretical shear stress values obtained using the method of van Driest.[2] The test section stagnation pressure ranged from 0.77 to 5.88 MPa. The flow Mach number was nominally 3.2. The relationship is linear, confirming the fact that shear stress has effectively been isolated from the unwanted contributions due to pressure, temperature, and flow-induced vibration. One of the major problems encountered in the development of the gauge was the decoupling of pressure and shear stress. For the range of conditions considered, pressure varies as a nonlinear function of shear stress. Hence, if the gauge were responding to pressure, the linear relationship in Fig. 3 would not occur. Using the nominal manufacturer's sensitivities for the piezoceramic material, calculations indicate that the gauge outputs for the theoretical shear stresses should lie along the second straight line in Fig. 3. This small difference is not surprising and further confirms the proper functioning of the gauge.

Table 1 Four test conditions

	1	2	3	4
Stagnation enthalpy, MJ/kg	12.07	9.507	7.34	5.76
Stagnation temperature, K	6348	5287	4373	3763
Stagnation pressure, MPa	5.88	3.62	1.63	0.77
Temperature, K	3383	2262	1952	1463
Pressure, kPa	114	65.8	27.4	12.1
Density, kg/m^3	0.107	0.079	0.046	0.027
Velocity, m/s	3486	3134	2789	2444
Mach number	3.10	3.13	3.20	3.22

Conclusions

Tests of a prototype skin friction gauge at Mach 3.2 in a small free piston shock tunnel have demonstrated the effectiveness of the design concept and the calibration against theoretical skin-friction values in a simple flow. The gauge has a rise time of about 20 μs, sufficiently short for most shock-tunnel applications and approaching the rise times needed for expansion tube applications.

Acknowledgments

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Large Deflections of Sandwich Plates with Orthotropic Cores—A New Approach

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Introduction

THE problem of large deflections of isotropic sandwich plates has been investigated by several authors.[1-6] Kamiya[5] presented governing equations for large deflections of isotropic sandwich plates following Berger's approximation. Accuracy of his solution depends on a correction factor. Dutta and Banerjee[6] have offered a simplified approach to investigate the nonlinear static as well as dynamic behaviors of sandwich plates. The literature on large deflection analysis of sandwich plates of orthotropic materials is scarce.[7,8] The present study investigates the large deflections of rectangular sandwich plates with a core as an orthotropic honeycomb-type structure. It is felt that this type of core corresponds more

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Table 1 Values of central deflection for various loads

Side ratio, b/a	W_o/h immovable edge: Present study	W_o/h immovable edge: Obtained from classical equation given in the Appendix	W_o/h movable edge: Present study	W_o/h movable edge: Obtained from classical equation given in the Appendix	Load function, $q_o a^4/Eh^4$
1	1.548	1.49	2.46	2.48	10
2	1.802	—	3.055	—	10
3	1.855	—	3.18	—	10

exactly to the behavior of actual sandwich construction used in industry. Numerical results obtained from the present study have been compared with those obtained by solving the classical equations given in the Appendix.

Analysis

First, we take a rectangular coordinate system X,Y,Z: X,Y in the middle plane of the core, Z in the thickness direction (positive upwards).

The total potential energy of the system is given by

$$\bar{V}_o = \bar{V}_o^f + \bar{V}_o^{-c} \tag{1}$$

where $\bar{V}_o^f$ is the strain energy per unit area of both isotropic faces proposed by Dutta and Banerjee[6] and $\bar{V}_o^{-c}$ is the strain energy per unit area of the orthotropic core due to shear.[8]

Executing the variational calculus so as to minimize the total potential energy in Eq. (1) of the present elastic system of the sandwich plates, we arrive at the following sets of differential equations.

$$I_1^{1m} = \frac{\partial}{\partial x}(u^u + u^l) + \nu \frac{\partial}{\partial y}(v^u + v^l) + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right) + \frac{\nu}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \text{const} \tag{2}$$

$$\left[\frac{Et}{2(1-\nu^2)}\left(\frac{\partial^2 r}{\partial x^2} + \nu\frac{\partial^2 s}{\partial x \partial y}\right)\right] + \frac{Et}{4(1+\nu)}\left(\frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 s}{\partial x \partial y}\right) + \left(\frac{\partial w}{\partial x} - \frac{p}{h}\right)G_{xz} = 0 \tag{3}$$

$$\left[\frac{Et}{2(1-\nu^2)}\left(\frac{\partial^2 s}{\partial y^2} + \nu\frac{\partial^2 r}{\partial x \partial y}\right)\right] + \frac{Et}{4(1+\nu)}\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 r}{\partial x \partial y}\right) + \left(\frac{\partial w}{\partial y} - \frac{s}{h}\right)G_{yz} = 0 \tag{4}$$

$$\frac{Et}{1-\nu^2}\left[2I_1^{1m}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) + \lambda\left\{\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]\nabla^2 w + 2\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2} + 2\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2} + 4\frac{\partial^2 w}{\partial x \partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]\right\} + h\left(G_{xz}\frac{\partial^2 w}{\partial x^2} + G_{yz}\frac{\partial^2 w}{\partial y^2}\right) - \left(G_{xz}\frac{\partial r}{\partial x} + G_{yz}\frac{\partial s}{\partial y}\right) + q = 0 \tag{5}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

For a movable edge, $I_1^{1m} = 0$. Here $r = u^u - u^l$, $s = v^u - V^l$, and G_{xz} are the shear modulii.

We consider the bending of a simply supported rectangular sandwich plate ($a \times b$) with constraints in-plane displacements at the boundaries.

For a simply supported rectangular plate, we assume

$$W = W_o \sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{6}$$

r and s are assumed as follows[5]:

$$r = r_o \cos\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{7}$$

$$s = s_o \sin\frac{\pi x}{a}\cos\frac{\pi y}{b} \tag{8}$$

Putting Eq. (6) into Eq. (2), I_1^{1m} is evaluated through integration over the plate as

$$I_1^{1m} = \frac{W_o^2\pi^2}{8}\left(\frac{1}{a^2} + \frac{\nu}{b^2}\right) \tag{9}$$

The in-plane displacements u^u, u^l, v^u, v^l of the upper and lower faces have been eliminated through integration by choosing suitable expressions for them, compatible with their boundary conditions. This is because the normal displacement "W" is our primary interest.

Putting Eqs. (6–8) into Eq. (5), recalling the values of r_o and S_o and the value of I_1^{1m} obtained from Eqs. (3), (4), and (9), we arrive at the following cubic equation determining the central deflection $W_o(x,y)$ after applying Galerkin procedure.

$$\frac{4\pi^4 Et}{ab(1-\nu^2)}\left\{\frac{(b^2+\nu a^2)^2}{16a^3b^3} + \lambda\left[\frac{1}{32ab} + \frac{9}{64}\left(\frac{b}{a^3} + \frac{a}{b^3}\right)\right]\right\}W_o^3 + \frac{\pi^2 W_o}{ab}\left[h\left(G_{zx}\frac{b}{a} + G_{yz}\frac{a}{b}\right) + G_{xz}\frac{bk_3}{k_2} - G_{yz}\frac{k_1 a}{k_2 b}\right] = q_o \tag{10}$$

where

$$k_1 = \frac{Et\pi^2(3-\nu)}{4a^2(1-\nu^2)}G_{xz} - G_{yz}\left[\frac{Et\pi^2}{4(1-\nu^2)}\left(\frac{2}{a^2} + \frac{1-\nu}{b^2}\right) + \frac{G_{xz}}{h}\right]$$

$$k_2 = \frac{E^2t^2\pi^4}{16a^2b^2(1-\nu^2)} - \left[\frac{Et\pi^2}{4(1-\nu^2)}\left(\frac{2}{a^2} + \frac{1-\nu}{b^2}\right) + \frac{G_{xz}}{h}\right] \times \left[\frac{Et\pi^2}{4(1-\nu^2)}\left(\frac{2}{b^2} + \frac{1-\nu}{a^2}\right) + \frac{G_{yz}}{h}\right]$$

$$k_3 = \frac{G_{xz}}{a}\left[\frac{Et\pi^2}{4(1-\nu^2)}\left(\frac{2}{b^2} + \frac{1-\nu}{a^2}\right) + \frac{G_{yz}}{h}\right] - G_{yz}\frac{Et\pi^2}{4ab^2(1-\nu)}$$

and the load "q" considered being sinusodial and given by

$$q = q_o \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$

Numerical results have been obtained for a rectangular sandwich plate of orthotropic core with the following data[5,7]: $a = 10$ in., $t = 0.025$ in., $h = 0.6746$ in., $E = 10.45 \times 10^6$ psi, $\nu = 0.03$, $\lambda = 0.09$, $G_{xz} = 10^4$ psi, and $G_{yz} = 10^3$ psi.

Table 1 shows the maximum deflection parameter (W_o/h) for simply supported rectangular sandwich plates with different side ratios, for the given load function ($q_o a^4/Eh^4$).

Conclusions

The results of the present study have been obtained with ease and accuracy whereas the solutions to the classical equation involves mathematical complexity and much computational labor. Thus, the differential equations proposed in the present study are simple because of its decoupled form and seem to predict the nonlinear behaviors of sandwich plates with orthotropic core with much ease and accuracy.

Appendix

Alwan[8] proposed differential equations for large deflection of sandwich plates with orthotropic core in terms of the Airy stress function. Now the displacement formulations of his proposed equation take the following form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{(1-\nu)}{2}\frac{\partial^2 u}{\partial y^2} + \frac{(1+\nu)}{2}\frac{\partial^2 v}{\partial x \partial y} = -\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} - \frac{(1-\nu)}{2}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2} - \frac{(1+\nu)}{2}\frac{\partial w}{\partial y}\cdot\frac{\partial^2 w}{\partial x \partial y} \tag{A1}$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{(1-\nu)}{2}\frac{\partial^2 v}{\partial x^2} + \frac{(1+\nu)}{2}\frac{\partial^2 u}{\partial x \partial y} = -\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2} - \frac{(1-\nu)}{2}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2} - \frac{(1+\nu)}{2}\frac{\partial w}{\partial x}\cdot\frac{\partial^2 w}{\partial x \partial y} \tag{A2}$$

$$\left(1 - D_y\frac{\partial^2}{\partial x^2} - D_x\frac{\partial^2}{\partial y^2}\right)\Delta\Delta\, W = \frac{1}{D}\left[1 - \left(D_y + \frac{2D_x}{1-\nu}\right)\frac{\partial^2}{\partial x^2} - \left(D_x + \frac{2D_y}{1-\nu}\right)\frac{\partial^2}{\partial y^2} + \frac{2D_xD_y}{1-\nu}\Delta\Delta\right]\left(q + \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x\partial y}\cdot\frac{\partial^2 w}{\partial x \partial y}\right) \tag{A3}$$

where

$$\frac{\partial^2 F}{\partial y^2} = \frac{2Et}{1-\nu^2}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \nu\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]\right\} \tag{A3a}$$

$$\frac{\partial^2 F}{\partial x^2} = \frac{2Et}{1-\nu^2}\left\{\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \nu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\right\} \tag{A3b}$$

$$\frac{\partial^2 F}{\partial x \partial y} = -\frac{Et}{1+\nu}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y} \tag{A3c}$$

F being the Airy's stress function.

Here u, v, w are displacement components along the X, Y, Z, directions, respectively, and

$$D_X = \frac{(1-\nu)D}{2hG_{xz}}, \qquad D_Y = \frac{(1-\nu)D}{2hG_{yz}}$$

For homogeneous plates, $D_x = D_y = 0$.

Let us now analyze the case of a simply supported square plate of side a. For this purpose, we solve first the equations for u and v. Let $W = W_o \sin(\pi x/a)\sin(\pi y/b)$, which satisfies the simply supported edge conditions. Putting the value of W in Eqs. (1) and (2), the solutions of u and v can be obtained in the following form:

$$u = Ax + C_1 \sin\frac{2\pi x}{a} + C_2 \sin\frac{2\pi x}{a}\cos\frac{2\pi y}{a} \tag{A4}$$

$$v = By + C_3 \cos\frac{2\pi x}{a}\sin\frac{2\pi y}{a} + C_4 \sin\frac{2\pi y}{a} \tag{A5}$$

where A and B are determinded from the boundary conditions.

For movable edges,

$$A = B = -\frac{W_o^2\pi}{8a^2}$$

and for immovable edge conditions, $A = B = 0$. Here C_1, C_2, C_3, and C_4 are given by

$$C_1 = C_4 = -\frac{W_o^2\pi(1-\nu)}{16a}$$

$$C_2 = C_3 = \frac{W_o^2\pi}{16a}$$

Putting $W = W_o \sin(\pi x/a)\sin(\pi y/b)$, values of u and v obtained from Eqs. (A4) and (A5), using Eqs. (A3a-A3c), we finally obtain the following two cubic equations after applying Galerkin's technique.

For immovable edges:

$$\left\{\frac{E^2t\pi^6h^3}{64a^8(1-\nu^2)}\left[75\left(\frac{1-\nu}{G_{xz}} + \frac{2}{G_{yz}}\right) - (31+20\nu)\left(\frac{1-\nu}{G_{yz}} + \frac{2}{G_{xz}}\right) + \frac{Et(h+t)^2\pi^2(33-16\nu)}{2(1-\nu^2)a^2hG_{xz}G_{yz}}\right] - \frac{Eh^4\pi^4(1+2\nu)}{16a^6(h+t)^2}\right\}W_o^3 + \frac{Eh^4\pi^4}{a^6}\left[1 + \frac{Et\pi^2(h+t)^2}{4a^2h(1+\nu)}\left(\frac{1}{G_{xz}} + \frac{1}{G_{yz}}\right)\right]W_o = \frac{h^2q_o}{2a^2}\left[\frac{h^2(1-\nu^2)}{t(h+t)^2} + \frac{Eh\pi^2(3-\nu)}{4a^2}\left(\frac{1}{G_{xz}} + \frac{1}{G_{yz}}\right) + \frac{E^2\pi^4t(h+t)^2}{2a^4(1-\nu^2)G_{xz}G_{yz}}\right] \tag{A6}$$

For immovable edges:

$$\left\{\frac{E^2t\pi^6h^3}{64a^8(1-\nu^2)}\left[75\left(\frac{1-\nu}{G_{xz}} + \frac{2}{G_{yz}}\right) - (31+20\nu)\left(\frac{1-\nu}{G_{yz}} + \frac{2}{G_{xz}}\right) + \frac{Et\pi^2(h+t)^2(33-16\nu)}{2(1-\nu^2)a^2hG_{xz}G_{yz}}\right] + \frac{Eh^4\pi^4(3+2\nu)}{16a^6(h+t)^2}\right\}W_o^3 + \frac{Eh^4\pi^4}{a^6}\left[1 + \frac{Et\pi^2(h+t)^2}{4a^2h(1+\nu)}\left(\frac{1}{G_{xz}} + \frac{1}{G_{yz}}\right)\right]W_o = \frac{h^2q_o}{2a^2}\left[\frac{h^2(1-\nu^2)}{t(h+t)^2} + \frac{Eh\pi^2(3-\nu)}{4a^2}\left(\frac{1}{G_{xz}} + \frac{1}{G_{yz}}\right) + \frac{E^2\pi^4t(h+t)^2}{2a^4(1-\nu^2)G_{xz}G_{yz}}\right] \tag{A7}$$

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